Addition through Place Value Sense

Adding multi-digit numbers can also be thought of as adding digits in each consecutive place-value. This way it looks a lot like multiplying multi-digit numbers.

Let's look at 296 + 468:

	hundreds	tens	ones		
	2	9	6		
+	4	6	8		
		1	4	Ones	6 ones and 8 ones equals 14 ones
	1	5		Tens	9 tens and 6 tens equals 15 tens
+	6			Hundreds	2 hundreds and 4 hundreds equals 6 hundreds
	7	6	4		

This idea uses place value to add the digits in each place. So first we add ones, then tens and then hundreds. In fact, it doesn't even matter in which order you add in the places. But this right-to-left movement shows how the partial sums move over to the next higher place value each step. This is exactly what happens later in the multiplication algorithm. So it is worth spending time on this if it comes up.

Subtracting over the Integers through Place Value Sense. For example, 466 – 298:

	hundreds	tens	ones		
	4	6	6		
-	2	9	8		
			-2	Ones	6 ones less 8 ones equals -2 ones
		-3		Tens	6 tens less 9 tens equals -3 tens
+	2			Hundreds	4 hundreds less 2 hundreds equals 2 hundreds
	1	6	8		2 hundreds less 3 tens less two ones

Note that this one can be solved much faster without an algorithm:
466-298 = 466-300+2 = 166+2 = 168. Voila!

How adding and multiplying algorithms are very similar

		10s	1s
		6	8
	+	5	7
1s		1	5
10s	1	1	
	1	2	5

		10s	1s	
		6	8	
	x	5	7	
		5	6	1s
	4	2		10s
	4	0		10s
3	0			100s
3	8	7	6	

Addition with decimal fractions is identical to addition with whole numbers:

		100s	10s	1s
		6	8	5
	+	5	7	9
1s			1	4
10s		1	5	
100s	1	1		
	1	2	6	4

		1s	0.1s	0.01s
		6.	8	5
	+	5.	7	9
0.01s			1	4
0.1s		1	5	
1s	1	1		
	1	2.	6	4

Multiplying with decimal fractions is identical to multiplying with whole numbers

	1s	0.1s		
	6.	8		
x	5.	7		
		5	6	0,01s
	4	2		0.1s
	4	0		0.1s
3	0			1s
3	8.	7	6	

	1s	0.1s	0.01s		
	6.	8			
x	0.	5	7		
			5	6	0,001s
		4	2		0.01s
		4	0		0.01s
	3	0			0.1s
	3.	8	7	6	

Subtracting with zeroes

Example 706-48

Traditionally the problem is that there aren't any tens to trade for ones. We then go to the hundreds (in the above case) and trade these for tens and then one of the tens for ones. This is not necessary if children learn to read and understand numbers more flexibly. So 706 can be thought of a 70 tens and 6 ones, 7006 can be thought of as 700 tens and 6 ones, or 70 hundreds and 6 ones, or 7 thousands and 6 ones. This flexibility is at the heart of what I propose below. One needs to establish this thinking first to make the method successful.

Decomposing the tens

706 can be thought of as 70 tens and 6 ones.
Decompose the 70 tens into 69 tens and 1 ten.
706 can then be thought of as 69 tens + 1 ten + 6 ones
We want to subtract 4 tens and 8 ones
This can be done by subtracting 4 tens from the 69 tens (yields 65 tens); subtract 8 ones from the 1 ten (this yields 2 ones) and add the additional 6 ones.

Numerically this looks as follows:

706	690	10	6		7006	6990	10	6	
-48	-40	-8			- 48	-40	-8		
	650	2	6	=658		6950	2	6	=6958

I also showed how it works exactly the same for 7006-48.

So maybe a good way to show this is to do a sequence of subtractions as follows:

76-48	(decompose 7 tens into 6 tens and 1 ten)
706-48	(decompose 70 tens into 69 tens and 1 ten)
7006-48	(decompose 700 tens into 699 tens and 1 ten)
70006-48	(decompose 7000 tens into 6999 tens and 1 ten)

Each time we decompose in tens, so that we can free up one ten to subtract the 8 from.

This then can provide a scaffold to trading the 1 ten for 10 ones, so that we have 16 ones less 8 ones, as we do in the traditional algorithm.

However, decomposing by tens is easier to do mentally. The subtraction then comes down to a fact of 10 (10 = 8 + 2).